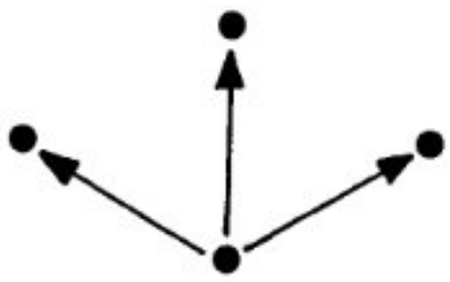
THE CLIMBING GAME

This game is for two players.

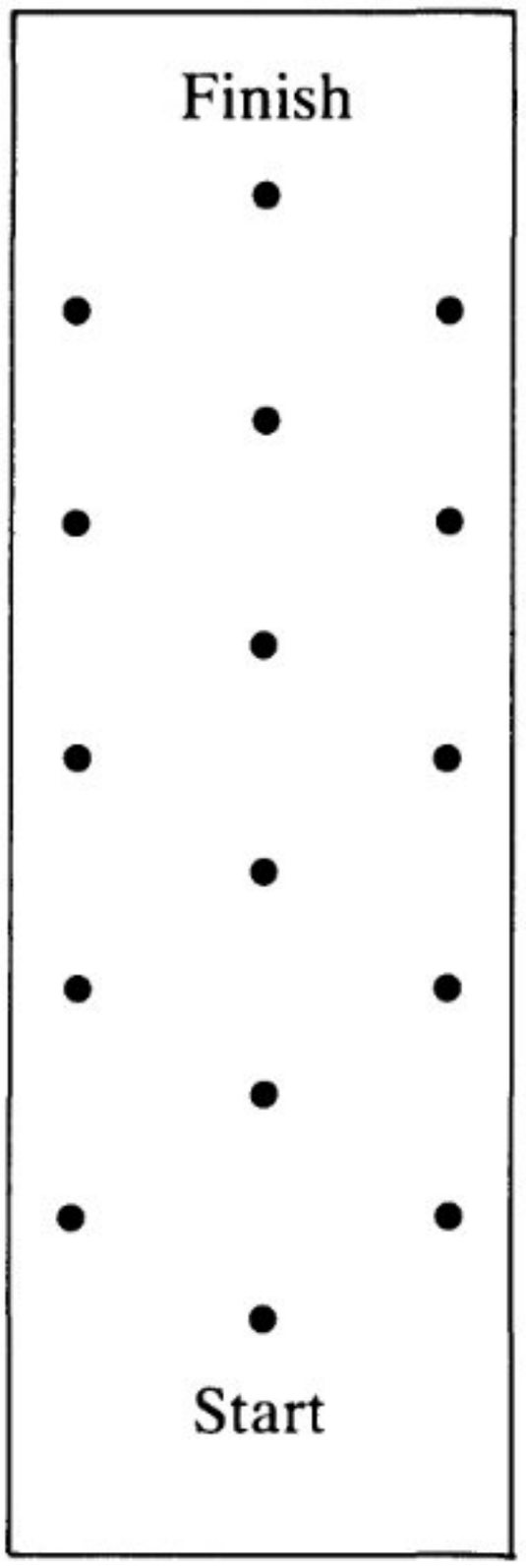
A counter is placed on the dot labelled "start" and the players take it in turns to slide this counter up the dotted grid according to the following rules:

At each turn, the counter can only be moved to an *adjacent* dot *higher* than its current position.

Each movement can therefore only take place in one of three directions:

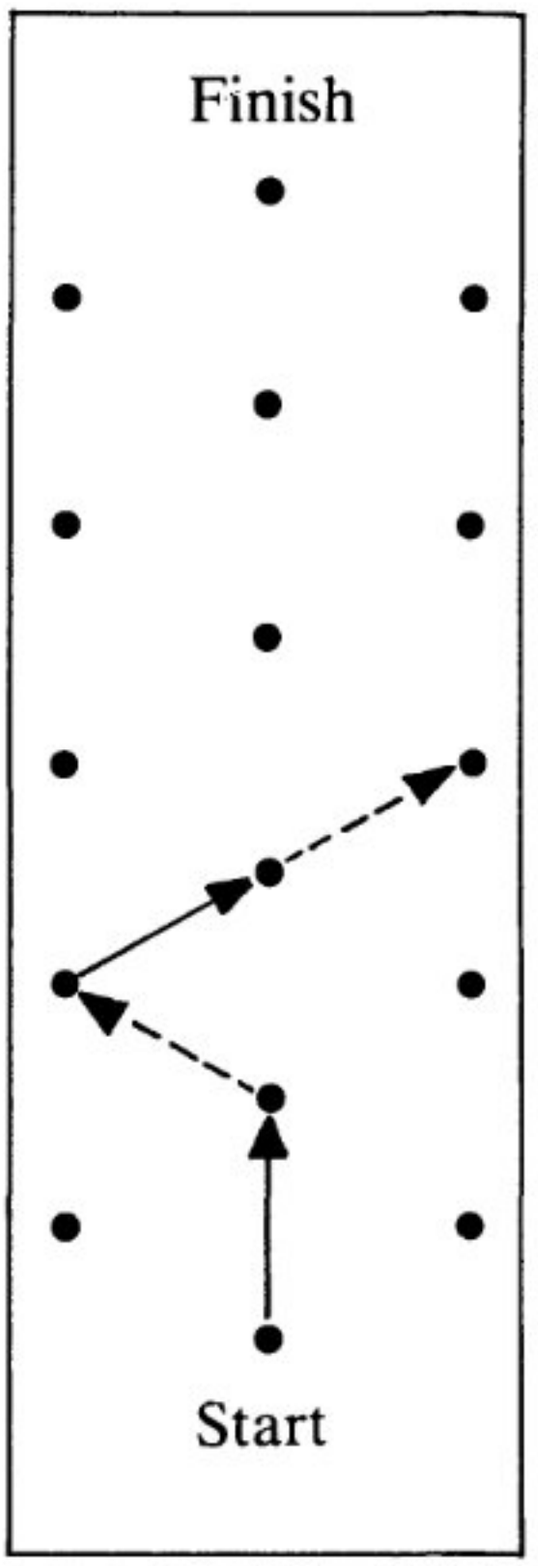


The first player to slide the counter to the point labelled "finish" wins the game.



(i) This diagram shows the start of one game, played between Sarah and Paul.
Sarah's moves are indicated by solid arrows (——→)
Paul's moves are indicated by dotted arrows (– – →)
It is Sarah's turn. She has two possible moves.
Show that from one of these moves Sarah can ensure that she wins, but from the other Paul can ensure that he wins.

(ii) If the game is played from the beginning and Sarah has the first move, then she can always win the game if she plays correctly.
Explain how Sarah should play in order to be sure of winning.



THE CLIMBING GAME . . . MARKING SCHEME

(i) **Showing an understanding of the rules of the game by systematically dealing with the various possible moves.**

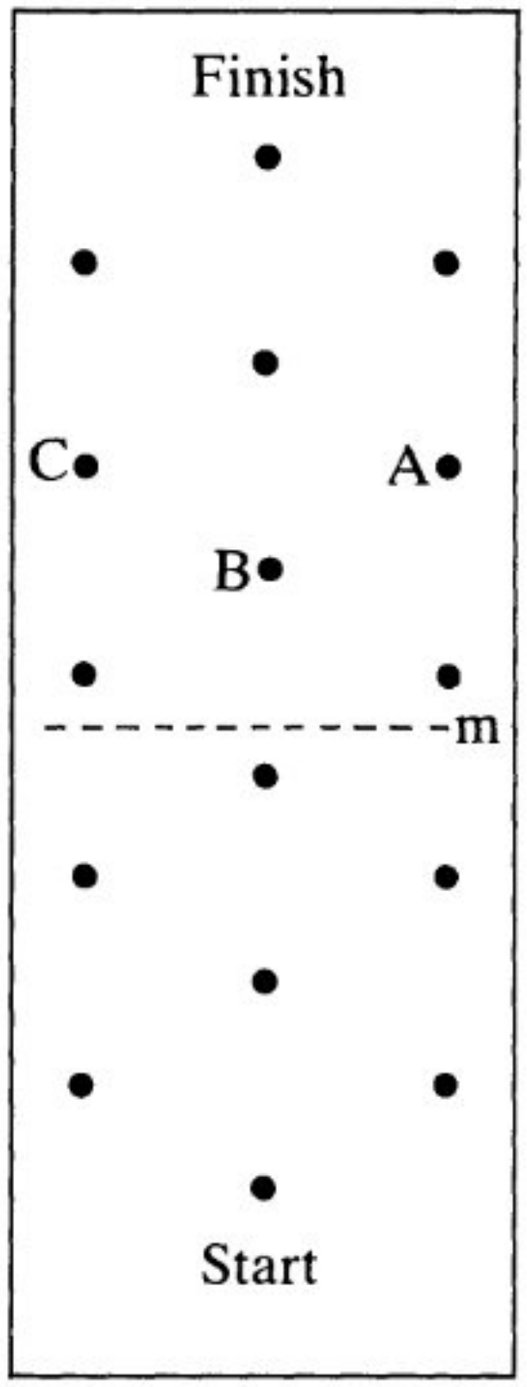
1 mark for indicating that Sarah can force a win by moving to point A or for indicating that she could lose if she moves to point B.

2 marks for a correct analysis of the situation if Sarah moves to point A including the consideration of both of Paul's possible moves.

Part mark: 1 mark for an incomplete or unclear analysis.

3 marks for considering the situation if Sarah moves her counter to point B and making a correct analysis.

Part marks: 2 marks for an analysis which is complete but unclear or which is clear but omits to consider one of the two possible moves for Sarah from point A or C. 1 mark for a more partial analysis.



(ii) **Formulating and explaining a winning strategy for the game**

4 marks for clear, complete and correct explanation.

Part marks: 3 marks for incomplete or unclear but correct explanation.

Up to 3 marks can be given for the following:

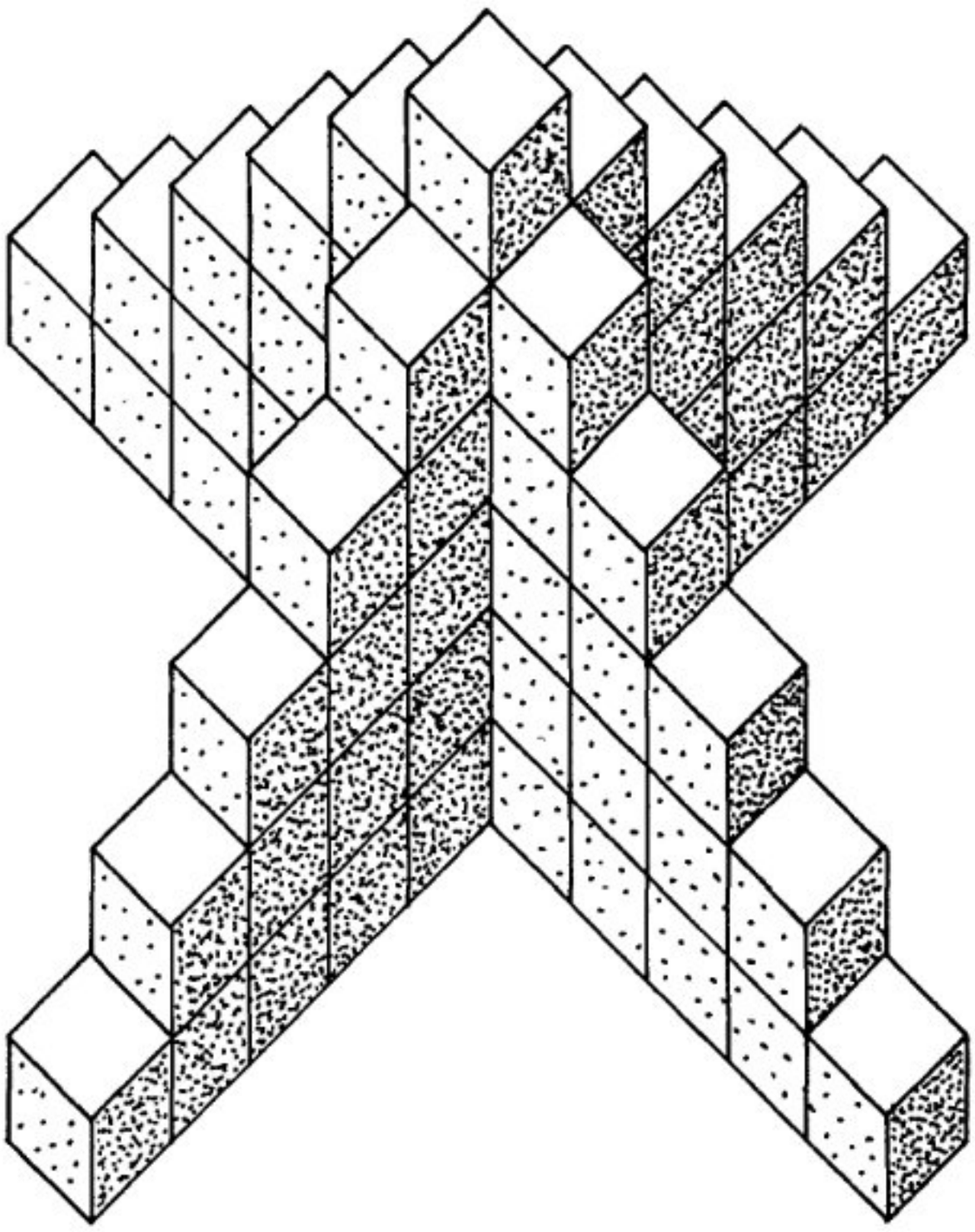
1 mark for recognition of symmetry.

1 mark for evidence of a systematic approach.

1 mark for correctly identifying some winning and/or losing positions above line *m*.

or 2 marks for correctly identifying some winning and/or losing positions below line *m* (or above and below line *m*).

SKELETON TOWER



(i) How many cubes are needed to build this tower?

(ii) How many cubes are needed to build a tower like this, but 12 cubes high?

(iii) Explain how you worked out your answer to part (ii).

(iv) How would you calculate the number of cubes needed for a tower n cubes high?

SKELETON TOWER . . . MARKING SCHEME

(i) **Showing an understanding of the problem by dealing correctly with a simple case.**

Answer: 66

2 marks for a correct answer (with or without working).

Part mark: Give 1 mark if a correct method is used but there is an arithmetical error.

(ii) **Showing a systematic attack in the extension to a more difficult case.**

Answer: 276

4 marks if a correct method is used and the correct answer is obtained.

Part marks: Give 3 marks if a correct method is used but the work contains an arithmetical error or shows a misunderstanding (e.g. 13 cubes in the centre column).

Give 2 marks if a correct method is used but the work contains two arithmetical errors/misunderstandings.

Give 1 mark if the candidate has made some progress but the work contains more than two arithmetical errors/ misunderstandings.

(iii) **Describing the methods used.**

2 marks for a correct, clear, complete description of what has been done providing more than one step is involved.

Part mark: Give 1 mark if the description is incomplete or unclear but apparently correct.

(iv) **Formulating a general rule verbally or algebraically.**

2 marks for a correct, clear, complete description of method.

Accept "number of cubes=$n(2n-1)$" or equivalent for 2 marks. Ignore any errors in algebra if the description is otherwise correct, clear and complete.

Part mark: Give 1 mark if the description is incomplete or unclear but shows that the candidate has some idea how to obtain the result for any given value of n.