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Designing for Mathematically Enriched Democratic Dialogues: The MinD Lesson Model

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Abstract

We describe a lesson model, recently developed as part of our MinD project (Math in Democracy), for designing mathematics lessons that promote engagement with civic deliberations. A core tenet driving the development of these lessons is that a mathematical lens is essential to understand aspects of many societal issues and that these mathematical elements should be part of the public discussion and deliberation about the issues. Such math-informed civic dialogues can lead to a fuller understanding of the issues, accounting for more perspectives, resulting in more fair and just actions. Another tenet driving our work is that education should contribute to the development of skills needed for engagement with civic dialogue and political decision making across different forms of government. In this article, we describe how the current MinD design principles were developed, through multiple iterations, from the Encounters program which uses a dialog protocol to engage a wide variety of community stakeholders in civic dialogue. We also share specific examples to illustrate the development process and analyze how the final lesson structures reflected design principles which adhere to the original elements of the Encounters dialogue protocol.

Introduction

How might mathematics and mathematical reasoning inform our civic discourse and lead to a more fair and just society? This is the question that animates some of our present work. It further prompts a compelling follow-up question: What then should be the role of mathematics education in supporting such civic discourse and reasoning?

In this paper, we describe a recently developed model as part of our Math in Democracy (MinD) project for designing mathematics lessons that support, and are supported by, civic dialogue. The MinD project promotes the idea that the teaching and learning of mathematics in schools should foster and develop students' skills to engage in deliberative dialogues about civic issues that draw on quantitative elements. With the recent rise of tensions and divisive political stances around the globe, and perhaps especially the United States, arguments are often formed and guided by emotions. The health and robustness of a representative democracy, however, requires civic dialogue and requires this capacity among its citizens (Council of Europe, 2018a; Lee et al., 2021; Stitzlein, 2020). Furthermore, we assert that this civic dialogue requires attention to quantitative reasoning. In the present day, we are steeped in quantitative information, and our

understanding of a situation and our choices is shaped by algorithms and processes (O'Neill, 2016; Wolfram, 2020). Mathematics education then has much to contribute to promoting a healthy and robust democracy (Ani, 2021) and a more just society (Berry et al., 2020) by allowing citizens to gain additional insights into an issue, and to learn about one another's concerns, values, and goals, which inform decision making. The MinD model was developed to reveal and engage the mathematical aspects of important civic issues, and to provide individuals the opportunities and tools to reason and voice their stances while respecting and understanding others' viewpoints.

Towards this end, we document the MinD lesson model, specifically, its origins, our revisions over time, and our working design principles for MinD lessons. We describe how the current design principles were developed, building from the Encounters program (Humanities for All, 2022), which centralizes a protocol process to engage a variety of community stakeholders in civic dialogue. We include examples, using pilots from our early lessons, to illustrate the process. We also show how one lesson (Connecticut's Educational Cost Sharing Formula) reflects the design principles we developed. These design principles necessarily adapt the structure of the Encounters dialogue protocol, but also remain grounded in the original key elements of the protocol.

In the concluding section, we connect this work to the (contested) purposes of education and how mathematics education can, and we contend should, support and promote a healthy democracy. We emphasize that there is a role for both the *content* of mathematics class, as well as the *practices* used to engage mathematics, in supporting students'/citizens' capacity to engage in civic dialogue and promote a more just and equitable society.

Conceptual Foundations and Prior Work

Researchers and theorists consistently call upon civic and educational organizations to promote thoughtful dialogue about issues that relate to the public good (Stitzlein, 2020). From Dewey (1923) through modern day, it has been asserted that an essential role of schooling is to develop a citizenry that can participate in and further its democratic institutions (Council of Europe, 2018a). Although exactly *what* one needs to know may vary over time and across specific forms of democracy, and may not have consensus, there is general agreement that such civic engagement requires students to learn to participate in civic discourse and reasoning (Brammer et al., 2012; Mallory & Thomas, 2003), a stance supported by sociocultural theories and situated perspectives of learning (Lave & Wenger, 1991; Rogoff, 1997; Wenger, 1999). There have been many efforts and initiatives to promote interactions aimed at engaging the public in civic discourse or expanding citizens' capacity for engaging such discourse. These initiatives reflect the assertion that one cannot learn to participate in civic dialogues without participating in civic dialogues.

In this section, we define some key terms and argue the need for a robust civic discourse, informed by mathematics, to advance a more just society. We review 1) some initiatives that aim to foster civic discourse across formal and informal organizations and 2) extant initiatives in mathematics education that engage and reveal to students how mathematics is a tool to frame and address societal issues and issues of justice. We conclude by suggesting the need for a lesson model that does both—grows capacity for civic discourse and uses math as a tool for analyzing and addressing societal issues. Our MinD lesson structure was designed to pursue both aims, which we then elaborate in subsequent sections.

Civic Discourse and Reasoning and Advancing a More Just Society

Civic discourse and civic reasoning are two terms used frequently in the literature. *Civic discourse* refers to dialogues among citizens about issues relevant to public life and decision making for the public good. (We use the terms discourse and dialogue interchangeably in this paper.) *Civic reasoning* refers to the nature of reasoning used in these dialogues to promote various viewpoints about the issue and potential course of action. Drawing on Stitzlein (2020) and Levine (2016), civic reasoning is what we do when we deliberate "*What should we do?*"

Civic discourse and civic reasoning are processes connected with promoting a more just society. Justice invokes ideas of fairness and moral actions. Justice can be defined as "fairness in the way that people are treated" (Collins Dictionary, n.d.) or "the condition of being morally correct or fair" (Cambridge Dictionary, n.d.). Groups of people engage civic discourse and reasoning—deliberating *what should we do?*—in preparation for action. This dialoguing creates opportunities to raise up and consider multiple perspectives and viewpoints. In this way, civic discourse and civic reasoning support healthy democratic processes where members of a community learn about one another's concerns, values, and goals and are better positioned to promote a fair and just action.

The value and impact of this civic discourse and reasoning is not limited to democratic societies. In non-democratic structures, there is still some consideration of people's views and voices, particularly when expressions, such as protests, receive international attention. For example, in recent years, we have witnessed, and governments have responded to, actions of some athletes and teams in the World Cup (Nikpour, 2022) and voices have been heard by those participating in the white paper protest in China (Rosen, 2022) where the act of not using words on a paper was a form of advocacy. Even when citizens cannot fully engage in all aspects of civic discourse and reasoning, and dialogue may be muted, the discourse promotes awareness of others' perspectives and conditions, and it can ultimately lead to more just policies and outcomes.

Articulating a Role for Mathematics in Civic Discourse

What is the role for mathematics in advancing a more just society? We overview frameworks and initiatives that reveal how mathematics is used to help us see, analyze, and act in relation to pressing societal issues, and even how math can create crises in society (e.g., Ani, 2021; Berry et al., 2020; O'Neil, 2016). We begin with the Council of Europe's framework, and then discuss two approaches that use mathematics (and mathematics education) to press toward a more just society. The first, Social Justice Math, focuses on using math to reveal injustices. The second, which we term informally societal uses of math, focuses on how mathematics is *used* by our societies, for collective planning and decision making.

Council of Europe Reference Framework of Competences for Democratic Culture.

The Council of Europe's (CoE) *Reference Framework of Competences for Democratic Culture* (2018a, 2018b, 2018c) offers a detailed set of standards and indicators describing competencies needed by citizenry in democratic governance structures to sustain democratic culture. Their Reference Framework reflects the assertion that our democratic institutions are only as strong as the democratic practices that support and animate them. Specifically, Volume 2 (Council of Europe, 2018b) outlines indicators and standards to be pursued with descriptors that "are intended to help educators design learning situations

that enable them to observe learners' behaviour in relation to a given competence" (p.7). The descriptors are not written for any one subject area but derive from an analysis of the demands of democratic life (Council of Europe, 2018a). Some competences have clear connections with content and skills typically considered mathematical. For example, three indicators identified with a strong mathematical connection are the following:

- Can assess society's impact on the natural world, for example, in terms of population growth, population development, resource consumption (standard 20. Indicator 121, p. 23)
- Can reflect critically on the risks associated with environmental damage (standard 20. Indicator 122, p. 23)
- Shows that he/she feels secure in his/her abilities to meet life's challenges (standard 8. Indicator 48, p. 18)

Each of these indicators implicates quantitative reasoning competencies and tools—not in isolation, but alongside other tools—that are needed for individuals to assess, reflect, and have the capacity to make sense of the world around them. Importantly, the CoE is clear that democratic norms that govern dialogue and institutions are needed to sustain democratic practices, and not just content knowledge or policy.

Social justice math.

Within mathematics education, a significant movement *math for social justice*, or *social justice math (SJM),* suggests a strong role for mathematics in advancing a more just society. SJM often engages students in mathematically analyzing or assessing a situation to examine issues of fairness (Evans & Staples, 2021) and bias, and to consider taking action (e.g., Bartell, 2013; Gutstein, 2003; Gutstein & Peterson, 2005; Kokka, 2019). Although SJM researchers and educators pay attention to the norms needed to govern classroom conversations and ensure they stay safe and productive, the focus is primarily on the tasks and contexts. A driving goal is to promote math as a valuable tool to *read and write* the world (Gutstein, 2003), and to foster student empowerment.

Societal uses of math.

Other approaches broaden the framing to consider the role of mathematics in society. Niss (1994) outlines compelling "specialized practice areas" to describe the uses of mathematics in society (beyond building the discipline itself).

> Mathematics is involved more directly in a number of *specialized practice areas*.... To mention just a few: prediction, decision-making and control in the *social sphere*; description and forecasting of phenomena and events in *segments of nature*, perhaps modified by man and society; utilization and allocation of *natural resources*, renewable or extinguishable; and design, operation and regulation of *industrial and socio-technical systems*.
> Niss (1994 p. 369)

These practice areas point to the extensive potential uses and impact of mathematics in society, with implications for collective decision making.

Focusing specifically on uses of math in relation to crises in society, Skovsmose (2021) discussed how mathematics is used in three important ways. He asserts mathematics can be used to *picture a crisis, constitute a crisis,* and *format a crisis*. The first two uses—picture and format a crisis—are common for SJM work and overlap with Niss's practice areas. Picturing and formatting help us see and understand a crisis, providing

opportunities to develop approaches to addressing the crisis. Skovsmose's third use – *constituting a crisis* – is well represented by Cathy O'Neil in her book *Weapons of Math Destruction* (2016) with her discussions and insight into the world of algorithms and big data and its potential harm (e.g., predictive policing).

This work collectively establishes the centrality of mathematics as a tool for advancing societal understanding of civic issues and finding effective ways to respond to the question, *what should we do?* Furthermore, it broadens the rationale for a strong mathematics education, which has generally been grounded in promoting a strong national defense and maintaining an economic edge in the global markets (Tate, 2013).

Developing Capacity for Math-Informed Civic Dialogue

We have established the importance of civic discourse in advancing a more just society, as well as this discourse to include attention to, or be informed by, mathematical tools. We turn attention to *how* one might go about developing the requisite capacity. That is, how might we pursue expanding capacity for a math-informed civic dialogue?

Within schools, attention to developing capacity for civic discourse has been encouraged by the use of specific models that promote civic dialogues and reasoning. Examples of existing models are the Harkness method (Hassan, 2015) and Hess's work on controversial public issues (CPIs) (Hess, 2002, 2008; Hess & Posselt, 2002). These approaches centralize student voices and ideas, and they encourage an active facilitation role for the teacher. Outside of the classroom, there are a variety of initiatives and approaches which draw on different models and strategies (Mallory & Thomas, 2003; see a generative list at https://guides.lib.uw.edu/engagement/discourse). Such approaches are designed to 'create space' for democratic dialogues such as at civic meetings (e.g., League of Women Voters) or other community settings.

Scholars in Connecticut have developed the *Encounters* program to support civic discourse about myriad issues. The Encounters program is a collaboration among the University of Connecticut's Humanities Institute, Hartford Public Library and the Amistad Center for Art and Culture at the Wadsworth Atheneum. The heart of the program is a face-to-face gathering that utilizes a protocol process to engage a variety of community stakeholders in civic dialogue (https://humanitiesforall.org/projects/encounters). Encounters events focus on discussing humanistic topics, such as affordable housing or voting rights, and use a structured protocol, implemented by trained facilitators, to promote respectful dialogue among participants. The goal is not to engage in debate, but to engage in a process of gaining new perspectives and understandings. The Encounters model has both a content focus, with a goal for learning about a topic (specific to the session), as well as process goals (consistent across sessions). We elaborate on this program and its protocol in later sections.

The above discussion has focused on various models, structures, and protocols that support *participation in* (and potentially advancing capacity for) civic dialogue. These models are generally content agnostic. Our argument, however, has been that *mathematics* must play a prominent role. Our review of the literature identified no models that supported participation in *math informed* civic dialogue. Some writings about Social Justice Math do offer a lesson format, but it is not aimed at structuring and advancing capacity for civic dialogue explicitly. As one illustrative example, a recent book by Berry and colleagues (Berry et al., 2020), *High school mathematics lessons to explore, understand, and respond to social injustice*, provides a template planner for social justice math lessons. The planner outlines a general structure of Introduction/Engagement,

Investigation/Exploration, Share and Discussion, and Taking Action (pp 36-37). This sequence reflects best practices in mathematics education, drawing on reform-minded lesson structures such as the 5-Practices Routine (Smith & Stein, 2011) and Launch-Explore-Summarize (Connected Mathematics Project, n.d.). The civic-oriented component is prompted by the choice of the problem context and by adding a Take Action phase to the lesson. Unlike the examples described above (e.g., Hess's CPIs, Harkness model), the SJM model does not structure the dialogue component; students' participation in the dialogue is not an explicit goal. These are not limitations of the approach per se; they are only limitations if the explicit aim is to support students in learning to participate in civic dialogue.

If quantitatively informed civic dialogue is the goal, what's the approach? An emphasis on context and disciplinary skills does not necessarily lead to a citizenry that uses its disciplinary skills and knowledge to inform decision making and ideas about *what should we do?* (Rudolph, 2014). We wanted to extend prior work and design with a focus on both the nature of student conversations and the role of mathematics in those conversations. We sought a pedagogical approach to promote the capacity to engage in civic dialogue and reasoning, seeing it as equally consequential as the mathematics and contexts engaged. We prioritized topics related to decision making. Deliberating a future decision maintains the focus on fairness and is action-oriented.

Our commitments led us to what was essentially our key challenge. How might we design lessons to both authentically incorporate and reveal mathematics while also designing to support civic dialogue and engagement? Given the prevalence of mathematics in civic issues, we felt confident we could identify appropriate topics. With the rarity of models of civic discourse in mathematics classrooms, we were less sure about designing lessons that effectively engaged students in both mathematical reasoning and civic dialogue.

The previously discussed Encounters model resonated with our goals. Encounters relied on a well-articulated protocol and thus provided a useful starting point. In the next sections, we detail the development of the MinD lesson model, as it originated from the Encounters model and evolved over time. Note that we did not lead our design work with math content learning goals. That is, we did not design a math lesson with a civic context to address specific math content standards. We return to this point later, discussing how the design and implementation of these MinD lessons promote learning mathematics that is aligned with state standards but not focused on addressing specific content standards or indicators.

The MinD Model: Overview of its Development

We describe the phases of development of this lesson model. Although we share our work here as if it unfolded in a linear fashion, there were overlaps in the phases and our work did not always proceed so cleanly. As we discuss the MinD lesson model development, we share examples, and then exemplify our current working principles with one of our MinD lessons.

The Mathematics in Democracy (MinD) lesson model was developed to support civic dialogues that deliberately leveraged mathematics concepts and quantitative reasoning in high school mathematics classrooms. Such dialogues not only demonstrate to students the potential role of mathematics in democratic participation and decision making, but also provide practice in engaging civic dialogues that create opportunities for people (students) to exchange viewpoints about issues related to the collective good and civic life.

These dialogues, built into the MinD lesson model, have two defining features. The first is their **structure**. The lesson is organized around specified "rounds" guided by norms of openness with a respectful exchange of ideas, where all participants have the opportunity to voice their opinion and hear others' views. The structure is not intended to prompt a debate but rather to support an understanding of, and even refinement of, one's own viewpoint. The assumption is that all students—through the process of articulating their views and hearing others' views—will arrive at a richer understanding of an issue and a refined understanding of their own stance and position.

The second defining feature is their **focus**. The dialogue pursues an answer to *What should we do*? in relation to civic issues where quantitative reasoning supports sense making about the future action without requiring consensus. Civic issues where we must ask *What should we do?* are numerous. Some examples include whether to fund a bond for a new library; whether ranked choice voting is a better method for our elections, or which voting method would be most fair; and what proportion of state school funding should be allocated to which districts.

For our MinD project, we ultimately articulated five principles for designing future lessons, which we elaborate below.

Principles of Design for a MinD Lesson

- Use the structure of *Encounters/Collaborative Conversations*
- Create key questions that support a consideration of others' viewpoints
- Select lesson content with *contextual* information
- Select lesson content with *quantitative* information
- Develop an activity that prompts sensemaking—including quantitative sensemaking —of key ideas

By *sensemaking*, we mean that we design in ways that do not assume students will process the information shared with them. Rather, we must design so that students are prompted to consider or manipulate the quantitative information in context—a point we discuss more later in the article. Thus, while the Encounters model remained the core of the lesson structure, drawing out the quantitative reasoning component required adjustments to the model and careful attention to the above principles. We show through the next section how each came about, and then summarize at the end.

Development Process

Phase 0: Foundations: Encounters and Collaborative Conversations

The MinD lesson model emerged out of a pursuit to utilize the protocol developed for the Encounters Program (https://humanitiesforall.org/projects/encounters/) that was discussed above. We admired and were inspired by the purpose and structure of the Encounters protocol and its success in supporting civic dialogue across stakeholders. It needed to be adapted, however, to work within a high school setting.

The Encounters model comprises: (a) a protocol with timed segments (organized as Rounds), (b) key questions, and (c) short readings (which can also be visuals, recordings, etc.), with implementation supported by (d) moderators and (e) an expert in a related content area. The overall structure of the protocol, while acknowledging expertise and including shared documents, decenters expertise and is process driven rather than goal

driven. The participants are authors of the ideas from which the dialogue emerges. The expert responds to questions posed about the topic (and does not lecture or tell). Figure 1 overviews the Encounters model.

Figure 1 – How Encounters Works

- *Encounters* brings members of the public and topic experts together to explore topics of interest through structured dialogue.
- Before the program, participants review short readings related to the topic being discussed. This shared prior knowledge grounds the conversation in sound information and gives people the confidence to speak up.
- A head moderator introduces rules of civil conversation and guides the program, which consists of several "rounds" of timed conversations among groups of 6 - 8 people.
- In round one, the moderator shows an excerpt from the shared readings and a prompt question. Each participant has 2 minutes to speak on the subject, with no interruptions. Others must practice listening.
- In the next rounds, the moderator again shows excerpts and questions and each group has an open discussion, using the listening and speaking skills they practiced in the first round.
- In the Open Forum round, each group works as a team to form a collaborative question, which the topic experts answer. This satisfies curiosity and demonstrates the value of bringing scholarship, context, and real-life experiences into discussions of challenging issues.

As an example, one Encounters event focused on voting rights. Through deliberately chosen materials (brief readings), and a facilitated protocol with structured “rounds” of discussion governed by table rules (norms and agreed-to procedures), participants gained new understandings about voting rights (past and present) and expanded their capacity for engaging in civic discourse by participating in the facilitated and scaffolded dialogue model.

The Connecticut (CT) State Department of Education had recently created a pilot protocol, *Collaborative Conversations,* derived from the Encounters protocol, to fit the constraints of high school classrooms. Figure 2 overviews the Collaborative Conversations model and shows the sequence, organized into Rounds. (In Figure 1, these Rounds were described specifically in the last three bullets.) Importantly, Round 1 asks *all* participants to share their ideas. Each person has uninterrupted, though limited, time to share their initial views. In subsequent Rounds, the participants have an open discussion, supported by additional materials and questions. The Open Forum (Round 4) has further conversation, prompted by the formulation of questions and an expert’s responses to the questions. Figures 1 – 3 are slides from a presentation on the Encounters program protocol, and the adaptation Collaborative Conversations, sponsored by CT State Department of Education & The Connecticut Democracy Center at Connecticut’s Old Statehouse (Stephen Armstrong, 2020, personal communication).

Figure 2 – Sequence of Rounds for Collaborative Conversation

- **Round 1 - Timed Conversation (Individuals Share)**
  - View the reading together
  - Look at the questions and reflect
  - Each person shares their thoughts for 2 minutes, uninterrupted
- **Round 2 - Timed Conversation (Collective)**
  - View the reading together
  - Look at the questions and reflect
  - Have an open conversation following the table rules for 10 minutes.
- **Round 3 - Forming Collaborative Questions**
  - Each group works together for 3 minutes to create collaborative questions to ask the experts.
- **Round 4 – Open Forum**
  - One student from each table shares their group's question and experts field them collectively for 10 minutes.
- **Takeaways**
  - What did we learn about conversations? About the topic we discussed? About ourselves?

Both protocols reflect active learning principles, where all participants have the opportunity to— and to some degree are expected to—share their ideas and perspectives and engage higher-order skills (e.g., creating, analyzing, and applying) rather than lower-order skills (e.g., memorizing facts) (Gibson et al., 2020; Krathwohl, 2002). The protocols further reflect core ideas of sociocultural learning theories that conceptualize learning as a change in participation and recognize that *how* one learns is tightly connected to *what* one learns. Thus, while participants are engaged in and learning about a specific topic, they are also engaged in and are learning about how to participate in democratic deliberations.

Figure 3 offers an overview of valued features of the model. This information is shared with participating students along with the table rules to help set the tone and norms for the conversation.

Figure 3 – Collaborative Conversations – key features of the high school adaptation of Encounters

Our goal is to adapt the *Encounters* model for high school students.

We think students and teachers will appreciate some of the model's main features:

- Rather than *telling* people how to have a good, polite, respectful conversation, *Encounters* provides opportunities to learn and practice conversation skills in a friendly environment.
- The model isn't focused on "winning". It's not a debate. There is no persuasion. It's about exploration and understanding.
- The program emerged from UConn's work on Humility & Conviction in Public Life.
- This approach acknowledges that it's fine to come to a conversation with convictions - but they should be tempered by humility.
- In other words - we should be humble enough to know that we can be wrong, that we can learn, and that listening to others will help us grow.

All elements of the Encounters protocol also apply to the Collaborative Conversations (CC). For CC however, the Rounds are shortened, and the teacher can serve as the expert.

The Collaborative Conversation model looked promising as a grounding for our work to bring quantitative reasoning to civic dialogues about *what should we do?* Our work in Phase 1 was primarily to determine if this endeavor would even be possible.

Phase 1: Initial Lessons and Pilots: Topics, Scope and Focusing Questions

The goal of Phase 1 was to create a pilot that offered an existence proof. To further understand the model and its key features, we talked to the leads on the Encounters and Collaborative Conversations (CC) projects; looked at examples of past Encounters events and their materials; spoke to a high school teacher who had been part of the CC pilot; and one author attended an Encounters event. With the CC protocol as the foundation, our initial efforts to create an existence proof focused on topic selection and materials (questions, readings). Our initial brainstorm included the following set of topics:

- Tolls in Connecticut
- Taxing Capital Gains
- Voting Methods (e.g., ranked choice voting)
- Gun Control/Gun Violence
- Vaccines
- Affordable Housing

Each of these topics had been in the Connecticut news in recent weeks or months and had a readily apparent quantitative aspect. We chose to prioritize topics that had a local impact, as we expected that would be more compelling for high school students. We also felt local topics would do more to advance towards goals of belonging and justice. Discussions of meaningful topics involve the self and coming to know others, as well as them coming to know you and your perspectives. When you are known in a setting, you are more likely to feel that you belong. Importantly, when a deliberation among people includes and honors more perspectives and ideas, the decision-making process is more likely to be just, as it connects and reflects many viewpoints.

CT Tolls Lesson Design and Pilot

We decided to work first on Tolls in Connecticut which was being discussed as the state debated how to raise revenue to invest in transportation infrastructure. Our selection was informed by the clear role for mathematics as well as our sense that this would be less controversial than other topics. As we were new to this work, we were concerned with our ability to manage a topic that might get heated—though we learned that, even with tolls, some citizens had very strong opinions!

To learn about the topic and the different points of view, we read from a variety of news outlets, identifying state-generated materials (for example, the governor had a plan he wanted promoted), and searched for policy-related documents that might talk generally or theoretically about tolls, pros and cons, etc. This background work informed our questions and materials selection.

Readers can find the initial materials that we developed at https://docs.google.com/presentation/d/14sQGPI_m_X4_I_610rpL2YjCgSZOjLs5. This pilot lesson's structure followed the Collaborative Classrooms protocol quite closely. For materials, we used a newspaper article outlining pros and cons of tolls (Five Reasons to Support Tolls and Five Reasons to Oppose them in Connecticut, Vigdor, 2019) for Round 1. The question in Round 1 was: "Which are the most compelling arguments pro and con for you? And why?" For Round 2, we used a message from the governor and a brief article from a news outlet on freight transportation (Lamb, 2019). The question for Round 2 was: "What initiatives would you vote for/support?" A social studies teacher who taught economics served as the 'expert' and a group of recently graduated pre-service teachers were the participants. The session was conducted online because of the pandemic and was followed by a debrief.

Prior to implementing the lesson, we (the authors) discussed what we thought "success" would look like: What should we consider a successful adaptation of, and implementation of, the CC model for our MinD purposes? We outlined the following (from meeting notes):

We consider the design of the materials "good" if:

- Sources are reliable, provide adequate context
- Sources offer some quantitative information relevant to the main topic – and that information informs the conversation
- If participants come to understand the topic in a new light
    - Others' views
    - Own views
    - New insight into the topic

Takeaways: CT tolls lesson design and pilot.

Based on feedback and our own reflections, we felt we had some success in designing this lesson using the math-informed CC model.

Successes: We were successful in raising up multiple perspectives, and participants noted it was a "different kind" of conversation. The participants also felt they came to some new ideas and understandings as they reflected on or took in new information. The participants felt that the quantitative information did serve a role in supporting the conversation. The numbers, in general, gave important contextual information for the discussion, and the statistics were relevant for understanding the asserted need for, and potential impact of, tolls. In addition, participants did find mathematics useful in helping to think about the impact on individuals (commuters vs non-commuters) and the fairness of that impact. Some questions raised were: Should those using the roads pay more? Do CT citizens already pay with our taxes, and tolls should be focused on commercial vehicles like larger trucks that pass through?

Not as Successful: There were two areas where we felt the tolls CC fell short of our goals. First, participants (as well as us) found that the topic was challenging to really get one's head around. There were so many factors at play, and—being in the midst of a pandemic with fewer people commuting and during a time when there was a press for more fuel-efficient cars—participants wanted more information in order to have a solid stance. Perhaps reflecting this, one participant felt there was perhaps a bit of a lack of closure. Thus, we felt we did not offer adequate context for the questions posed, or the questions were too broad.

Second, there were so many ways to think about the topic, and the economic aspects became so complex so quickly, it was hard for participants to feel any sense of confidence or commitment to their assertions. One could focus on trucks or cars; in-state or out-of-state; commuters or infrequent users; or the impact of tolls on diverting traffic off the toll ways. Without more information about how much revenue might be generated, how much it would cost to set up the system, the impact on various groups, and other alternatives for investing in infrastructure, it was difficult for participants to find a firm footing for the conversation and identify a choice that felt compelling to in response to *what should we do?* One participant offered that there was a need for benchmark numbers (e.g., volume of cars on roads) in order to make sense of some information given. We all left the discussion with lots of questions and wanting to know more about many areas. That can be a good outcome, as it shows learning and new areas of interest. That acknowledged, we wanted the math-informed CC experience to offer participants the opportunity to see how math not only provides insights on a topic, but helps them with decision making in public life, which was not the experience they had.

Takeaways: For the design work, we identified important takeaways from the session. First, the Encounters/CC lesson model was viable. Second, the question being posed needed to be more focused than "What initiatives or proposals would you support?". Third, we needed to pay attention to how participants would be supported to make sense of the quantitative information. Reading alone might not be sufficient. Processing the numbers in the information—quantities and statistics—might need additional context.

We also concluded that, though the general model seemed promising, this choice of topic (tolls) was not particularly interesting for students. It wasn't clear that students would engage in sustained analysis to decide their own views on the topic and might need something more compelling.

Vaccines Lesson Design and Pilot

With our existence proof behind us, our second early effort was a pilot of a CC about vaccines. We had thought about this prior to the pandemic—as information about measles vaccination rates and instances of measles in Connecticut and elsewhere had hit the news. With the pandemic, we felt it was even more important to look at this topic and we felt confident it would be engaging.

The questions

We spent a long time during the development phase for this lesson creating the questions. With the heated dialogue around vaccines and mask mandates (even though a vaccine did not exist at this point), we wanted to be sure that we put participants in a space where they could dialogue about the issues and engage multiple viewpoints. A dialogue focused on a personal stance would be unlikely to produce a sense of belonging or consideration of just action, and it might focus on individual's preferences and rights. Having a group pursue *what should we do?* meant that the key question we asked had to move beyond the individual and prompt participants to consider action that impacted everyone.

We developed the following two questions to focus the lesson: "What should the role of State Departments of Health (or the CDC) be in managing infectious diseases?" and "Under what conditions should a state government mandate vaccination?". Similar to the tolls session, we piloted it online, this time with a mix of pre-service and in-service teachers. Our expert was a school nurse. Here is an early version of those lesson materials (https://drive.google.com/drive/u/2/folders/1yqHnzr0YeVXnf6L7v70XQw8HDzev5-62).

We draw the reader's attention to the nature of the focal questions. We did not ask whether individuals were for or against something, as we did not want to focus on issues related to individual's rights. Rather, we asked them to think about a principled approach for a government body that could work across a range of instances of infectious diseases. In this way, we hoped to position participants as people living in a collective with consideration for the role of a government agency. We chose to offer some historical context about vaccines, and also information about a (relatively) recent measles' outbreak, to help frame the session.

The mathematics

We started with a focus on the measles, and then looked more broadly at vaccines for a wide variety of infectious diseases (mumps, rubella, whooping cough, etc.). The math focus was not singular. Mathematics was relevant for participants to consider the resources that were used to respond to an outbreak, to understand vaccine efficacy, and finally, to make sense of the various measures relevant to the impact of an infectious disease, specifically *mortality* and *contagiousness*.

One design change from our first pilot (tolls) was that we were more deliberate in introducing quantitative ideas that we evaluated as highly relevant for discussing the question *what should we do?* For this topic, we thought the concepts of efficacy of vaccines, contagiousness of a disease, and deadliness—all quantities which vary from disease to disease—had strong relevance for discussing policies of mandating vaccines. For example, low vaccine efficacy, contagiousness, and mortality, would not seem to warrant a vaccine mandate, whereas higher mortality would press more toward mandating, but would interplay with contagiousness, vaccine efficacy, and, of course, values and beliefs. While we wanted these concepts/quantities to be available to participants for them to bring into the conversation, we were concerned that a direct introduction might be too strong a move. We did not want participants to feel they had to use those ideas in conversation, which we thought would be likely given the norms of a high school classroom. Thus, we felt a tension between identifying quantitative information to consider and allowing participants to bring their ideas to the conversation. We chose to steer in the direction of vaccine efficiency, contagiousness, and mortality, as we felt like this adjustment was warranted for the purposes of promoting productive discussion that attended to quantitative aspects of a civic issue and allowed for different perspectives.

We used an interactive graphic that showed contagiousness and deadliness of many diseases, which was accessible and engaging to the group (The MicrobeScope at https://www.informationisbeautiful.net/visualizations/the-microbescope-infectious-diseases-in-context/). We shared the idea of vaccine efficacy directly by presenting the definitions and an example of vaccine efficacy on a slide. (The information was adapted from materials we had found about vaccine efficacy, but they were not particularly user friendly, so we did not include them as readings.)

Takeaways: Vaccines Lesson Design and Pilot.

This pilot implementation overall went well—with high engagement and interest—perhaps in part because of the current pandemic. From our post-session discussion with participants, feedback survey, and our design team's reflections, our design principles and choices were productive. Participants felt the session provided general information about vaccines, for example, the fact that there are about 12 mandated vaccines for school children in the state. It introduced important, relevant quantities like contagiousness and deadliness. Participants explored a variety of diseases and their features.

However, there was little evidence that participants inquired with the vaccine efficacy information or that the quantitative information about vaccines had informed their discussion or views. A question arose at this point that we had not previously considered. As much as the protocol was an affordance for supporting democratic dialogue, it also was a constraint. Mathematical concepts can be challenging to make sense of and understand. In this case, the idea of vaccine efficacy is not trivial. We wondered: if key mathematical concepts are needed to make sense of an issue or idea, are there opportunities within a CC protocol to learn new concepts, particularly quantitative? Is there only room to use known concepts? What if new concepts are crucial?

Further reflection on this point led us to think that the fact participants didn't inquire with math was reasonable, as the CC protocol was designed to be fairly structured and there was not space/time allocated to do that kind of inquiring, "fiddling" work. When discussions were happening, one could think and process ideas and viewpoints, but that can be quite different from playing with some mathematical ideas, comparing quantities, seeing the impact of adjustments in one quantity on another, doing some informal modeling and inferential work, etc. For example, for the vaccine session, it would have been interesting to explore how vaccine efficacy and mortality worked together. What's the actual impact on life for a given vaccine efficacy, with a given mortality rate? Would exploring that relationship influence someone's thinking about whether a state should or should not mandate a vaccine? What about the contagiousness of the disease? It seemed that knowing about these concepts, and also making sense of the consequences of these factors for a population, was needed for articulating or understanding a policy proposal and formulating one's views about the appropriate course of action. It also would be required for generating *just* proposals, which accounted for multiple viewpoints. This attention to inquiry with math drove our next revision.

Phase 2: Revision to Enhance Mathematical Work – Adding a "Math Task"

The third math-informed CC lesson we designed focused on the Educational Cost Sharing (ECS) formula for Connecticut. The ECS formula is Connecticut's approach to supplementing local district funding for schools with state funds. In the United States, local taxes typically support the schools. This approach to funding, however, can create inequities due to differences in district wealth and differences in needed resources, which can vary with demographic factors (e.g., large populations of English language learners; areas of concentrated poverty). All states have a system to distribute state-generated revenues to support local schools (Fischer et al., 2021). Each state's distribution of money reflects what it thinks is required to provide students an adequate education.

Connecticut's formula is based on the students' needs and the district's capacity to generate funds. Understanding how public education is funded in one's state is essential to evaluating the fairness of the approach and weighing in on any proposed changes, or even advocating for changes.

ECS lesson background.

We offer more details about this third lesson, as we later use it to illustrate some of the principles of a MinD lesson.

Connecticut's ECS formula is spelled out in state law, and the final allocation for each district is calculated and made public using a spreadsheet. Figure 4 shows the formula as shared in a report from the New England Public Policy Research Center (Zhao, 2021).

Figure 4 – Summary panel of the Educational Cost Sharing formula (Zhao, 2021, p.5)

The current Education Cost Sharing (ECS) formula is structured as follows:

ECS target aid = *foundation* × *total need students* × *base aid ratio* + *regional bonus*.

Foundation is intended to represent the cost of educating a typical Connecticut public school student who has no additional learning needs. It is currently set at $11,525 per pupil. *Total need students* is a weighted student count. If a district's low-income students account for more than 75 percent of total enrollment, its *total need students* is calculated as:

total need students = *enrollment* + 30% × *low - income students* + 15% × *English learners* + 5% × (*low-income students* - 75% × *enrollment*).

Low-income students are defined as students who are eligible for free or reduced- priced meals or free milk. For a district where low-income students account for less than 75 percent of total enrollment, the calculation of *total need students* excludes 5% x (*low-income students* - 75% × *enrollment*).

Base aid ratio represents the share of total education cost that is funded by the ECS grants. It is calculated as:

$$1 - [70\% \times \frac{\textit{town ENGL per capita}}{1.35 \times \textit{median (town ENGL per capita)}} + 30\% \times \frac{\textit{town median household income}}{1.35 \times \textit{median (town median household income)}}]$$

ENGL represents equalized net grant list, which is the full fair market value of taxable properties. Finally, *regional bonus* is awarded to members of regional school dis- tricts, with $100 per regional-school student scaled by the ratio of the number of grades in the regional district to 13.

The focal question we chose for this ECS MinD lesson was: *Is this fair?* That is, is this approach to distributing funds to school districts in CT fair? We chose this focus because fairness is at the heart of conversations about *what should we do* in a democratic society (Evans & Staples, 2021). Fair actions are just actions. Addressing this question requires making sense of the formula, which is not trivial.

There were two major aspects that we felt were crucial for making sense of, and weighing in wisely, on the fairness of this formula: (1) the factors taken into account (e.g., # of students, # of English learners, a measure of the town's capacity to raise funds) and (2) the weight or influence of each of those factors on the total amount. Having made sense of it ourselves, we knew it took intellectual work and discussions to get to a point where one could reflect on the formula—its components and weights—and consider whether it seemed fair.

Modifying the CC protocol to include a math task

Our original design, following the CC model, offered materials and allowed participants to make sense of the ideas as they chose. That approach, however, had not accomplished our goal of using mathematics to make sense of an issue and inform one's stance on *what we should do*. Our modification in this phase was a major adjustment to our design. We decided to explicitly prompt more active sense making and agency with respect to the quantitative aspects of the topic. This goal followed from the need to unpack and critique a formula for this particular topic as well as our desire to prompt more inquiry with mathematics more generally—which was not clearly present in the first two lessons. This adjusted design for our ECS MinD lesson attempted to provide a more direct opportunity to reason quantitatively about the topic.

In our first pass at revising the ECS lesson design, we had participants brainstorm factors they thought should be taken into account to determine the distribution of state funds in Round 1. We then explained the ECS formula and gave examples. This "directly explain it" approach was similar to the vaccine efficacy approach, where we relied on presentation and explanation to access a quantitative component, which ultimately was not a powerful way to help participants make sense of the formula.

In our next revision, we further adjusted Round 2 to provide more sense making and agency. In the CC model, Round 2 typically added "food for thought" with a reading(s) followed by table (small group) discussions. Our goal was to keep a focus of Round 2 on enriching the conversation, but we had a new way to go about this. We gave each group data from one district—Districts A, B, and C, which were real districts, but with no names —and asked what portion of $79M they felt their district should get. The $79M was the actual combined amount allocated to those three districts based on the ECS formula for the previous year. The data we provided students about their assigned district included factors such as percent of students in low-income households; number of students; number of school buildings; percentage of students classified as English learners; percentage of minority students; and number of teachers. Some of these factors were part of CT's formula and some were not. Participants could choose which factors they felt should influence ECS formula and the distribution of the state funds, depending on what they thought was relevant and important. Participants also could play with the data, calculating percentages, relationships between two or more factors (e.g., number of teachers and number of students), etc., to *inquire with* mathematics and gain new information and insights. This adjusted approach for Round 2 prompted much sensemaking activity with respect to the formula and quantitative information.

While pleased with the result, a new difficulty emerged. Our activity seemed to promote a debate rather than dialogue. Groups identified so strongly with their assigned district, that the question of *what should we do?* focused the "we" on their district, rather than thinking about a fair system for the state. Such actions are not just, and the feeling in the room was more of competition and less collaborative. This violated an original tenet of the Encounters model and CC protocol, and we could feel the difference. In addition, because each group only received information about their district, they were not provided enough information to support careful thinking about why their district deserved a certain portion of the money from the state. For example, if a district had 20% of its students receiving Free or Reduced-Price lunch, was that unusually high, low, or typical? Did it warrant extra funds? These data did not have enough context for participants to address the collective and questions of fairness.

In subsequent iterations, to offer more context, we provided state-level data on the same set of factors we provided at the district level (e.g., median income for a household, percent of CT students who were English learners). This adjustment supported each group's/district's choices and reasoning.

We also added a second share out for this activity. There was an initial share out, when each group/district offered their ideas about how much they should get from the $79 million in funding. After the groups heard from one another and learned about some specific data from other groups, they had the opportunity to propose a new amount and/or a new rationale for the amount. This second share out prompted groups to consider their own and other perspectives and situations, as well as discuss *what* information they felt was compelling and articulate *why* a factor mattered.

We made a final innovation: we gave the students the data as "cards" and not in a chart/table. This adjusted format allowed them to physically position factors and data as information they thought was important and move to the side the cards with the information they felt was less important. A version of this lesson is available here (or see Seeto et al., 2022).

Takeaways: ECS lesson and design.

This adjustment—designing for a sense-making activity to provide contact with the mathematics of the issue—proved to be a critical and significant innovation for our MinD lessons. In addition, this lesson design experience with the ECS formula made us more cognizant of our principle to include appropriate context. Often the readings had provided appropriate context. We needed to ensure students had enough context to discuss the issue and to make sense of any numbers and quantities they needed to work with. With this lesson design that had no upfront readings (a change made as many schools had moved away from graded homework), our first iteration was missing that component, and students' knowledge of the differences across districts and the overall state context, varied greatly.

Phase 3: Solidifying Current Working Principles of MinD Lessons

In Phase 3, we developed a fourth lesson. It started as a lesson on *Reparations for Black Americans* and later became *Bruce's Beach: Exploring Intergenerational Wealth*. The lesson highlighted the experiences of the Bruce family and the impact of waterfront property being taken from them in the early 1900s in Manhattan Beach, California (Xia, 2020).

As with the ECS Formula lesson, the incorporation of a math task was crucial for this lesson. The math task allowed participants to explore the element of time, and how time compounds losses and influences the accumulation of wealth. Though we had learned a lot from our previous design processes, we repeated some of our earlier mistakes in our first version of this session. We do not discuss those in detail but note it as part of our process that helped us solidify our current working design principles for a MinD lesson.

We turn now to reflecting on this journey and revisiting the design principles.

Reflection on the Journey and Highlighting the Principles

Our revised MinD model evolved to comprise Rounds that were modified from the original CC or Encounters model. Essentially, the MinD model Rounds 2 and 3 replaced rounds where additional materials were read and discussed during the session. These newly focused rounds had the same spirit and overall purpose—adding more "food for thought." What was different was *how* that was accomplished. We now directly organized working with the quantitative ideas and information in a designated segment. This structure provided participants the opportunity to explore and make sense of some of the quantitative aspects and the relationships between and among the different quantitative elements, as well as their consequences. Our design journey consisted of learning how to effectively create space for participants to engage mathematical elements of questions of civic import. We note that participants were given agency in this work. We were not presenting mathematics and asking them to apply the concept. Rather, we were presenting a problem situation and information and asking them to inquire with mathematical tools, concepts and ideas they had at their disposal.

In Figure 5, we track the adjustments, mapping our changes within each Round. Recall Figure 2 showed the original Rounds for Collaborative Conversations. We did not then change the overall structure of the protocol, but did change the nature of the components within the Rounds. In Figure 6, we show the final MinD model in terms of the progression of Rounds.

Figure 5 – Tracking change of adaptation from Collaborative Conversations to the MinD model

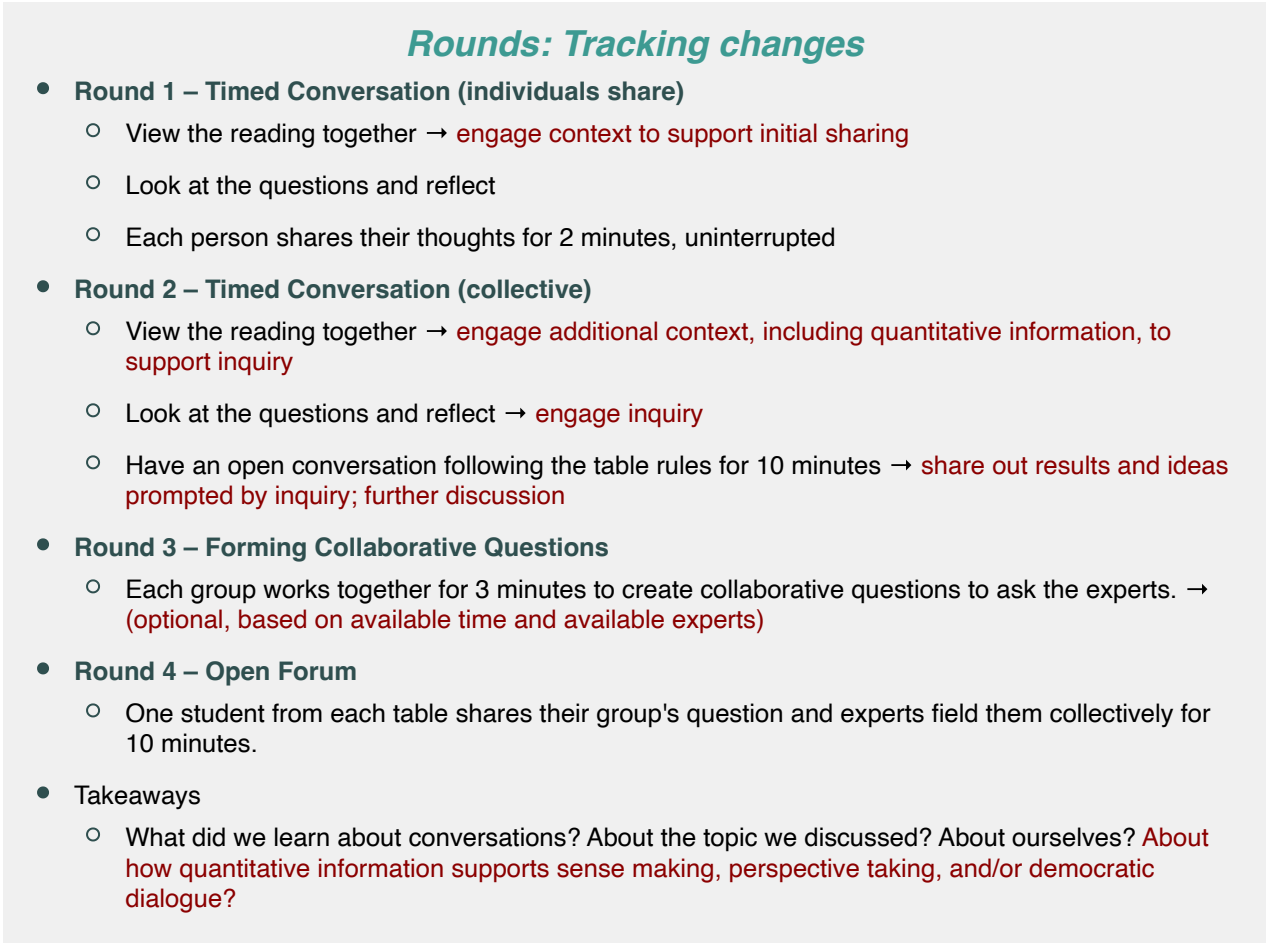
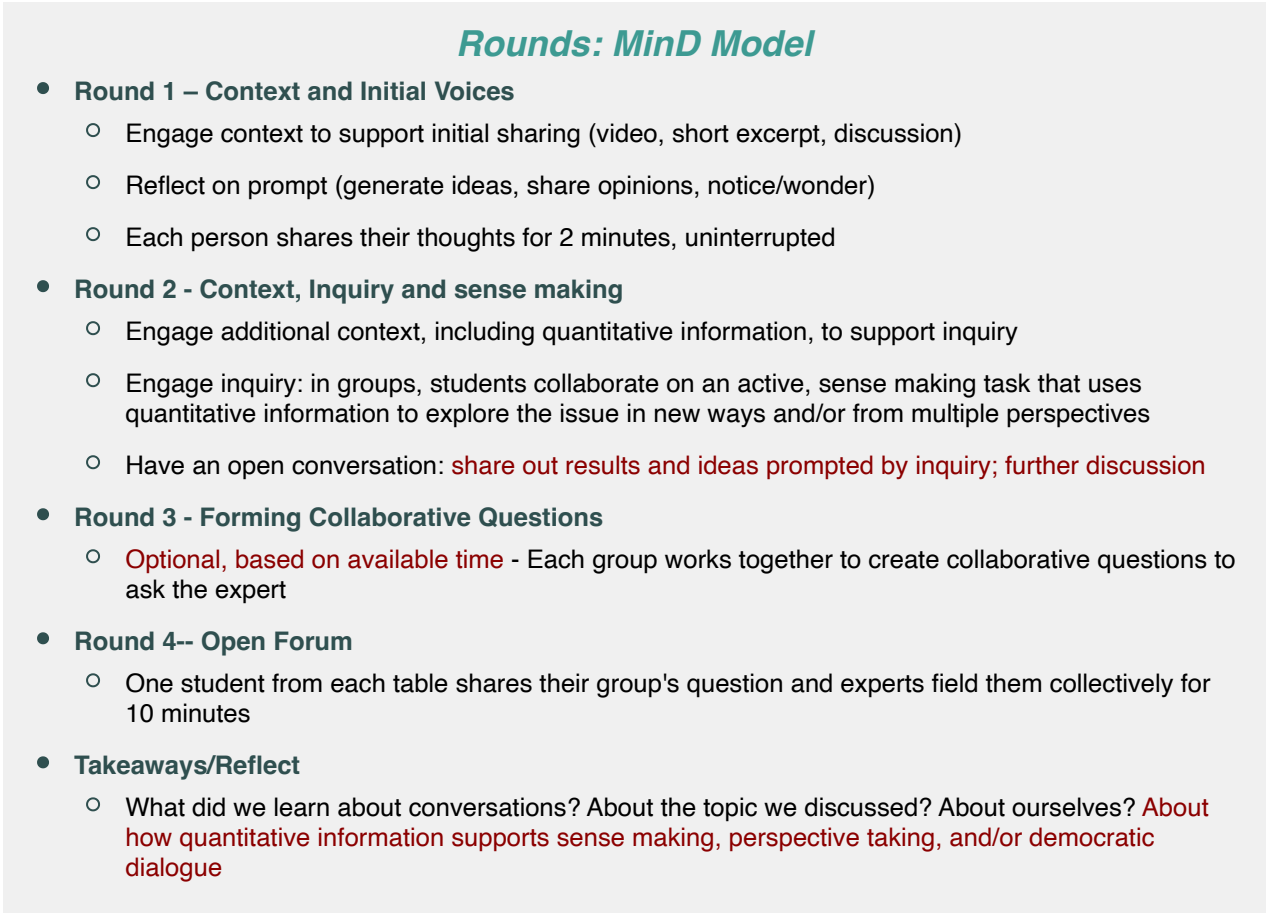


Figure 6 – MinD model Rounds and structure



To further summarize and synthesize the work shared, in Table 1 we articulate our design principles along with features that support that principle, and the impact of that principle in relation to our goal of promoting *math-informed democratic dialogues*. In the final column, we share how each feature was instantiated for the ECS MinD Lesson to help solidify the connections.

Table 1. MinD lesson Design Principles, Features, Impact and Examples from ECS lesson

Principle	Features	Impact/Value	Examples - relevant features in ECS MinD lesson
1. Use the structure of Collaborative Conversations/ Encounters	• Timed rounds • Table rules/norms • Readings • Opportunity to engage an expert	• All voices heard; all voices valued; norms of sharing and listening • Multiple rounds of sharing personal ideas • Readings/activities give information to support discussion; unfold complexity • Position as ongoing/continuing dialogue	• Used 4-round MinD lesson structure • Invited the state representative as the "public expert": positioned topic as needing ongoing inquiry; positioned students as active citizens.
2. Create key questions that support a consideration of others' viewpoints	• Key question(s) do not have a right answer, and are likely not a yes/no prompt • Key question(s) suggest consideration of *What should we do?* • Key question does not set up a debate, or focus on the individual	• Opens space for civic dialogue, where the individuals are invited to consider other perspective(s) and collective perspective • Presses students to analyze and interpret and integrate with their personal values • Through considering other viewpoints , one's own viewpoint is refined	• Key Questions • What factors would you consider when distributing money to different school districts? • Do you think the current ECS formula is fair?
3. Select lesson content with *contextual* information	• Content provides background information needed to engage and reason about the question(s) • Content offered in a consumable way (e.g., short readings, video clip, charts)	• Can introduce important background ideas/concepts useful for making sense of the issue • Supports dialogue with a shared foundation, as high school students vary in their knowledge of real-world issues • Supports reasoning about the civic issue	• Appropriate contextual information about *how* funding for CT schools was provided (e.g. sources and uses of funding) provided focus for discussion of distributing the cost-sharing funds
4. Select lesson content with *quantitative* information	• Content includes quantitative information relevant to civic issue • Anchor or benchmark information is provided to help make sense of relevant values and quantities. • Relevant quantities are defined as appropriate • Content offered in a consumable way (linked to principle 5)	• Can introduce important quantitative ideas/concepts that are useful for reasoning about civic issue • Supports dialogue with a shared foundation, as high school students vary in their knowledge of real-world issues • Supports reasoning about the civic issue	• Appropriate quantitative contextual information (variation, averages) about state and other districts helped participants make sense of the values (dollar amounts, number of people, percentage of populations) • Participants discussed which quantities and which weights they felt were fair and why
5. Develop an activity that allows for sense making – including quantitative sense making - of key ideas	• Quantitative information engaged in a consumable way • Activity is bounded, but open, offering opportunities for exploration and sense making • Sharable product from inquiry (visually accessible) to support further dialogue	• Prompts new ideas, insights and perspectives • Allows participants to share 'products' (or thinking generated by the products) • Potential to reveal value of quantitative information for making sense of a civic issue	• Provided cards with different (possible) relevant information and asked which are (most) relevant for the distribution of funds • Groups propose how much funding should be allocated from the $79M to their district (quantity plus rationale); groups hear others' views; can revise their proposals and rationale

Discussion

The Protocol

Our design process, which began with a well-established protocol to support democratic dialogues, required key adjustments to prompt *quantitative reasoning and civic reasoning*. Our adaptations for the MinD lesson model maintained design features from Collaborative Conversations that supported discourse about civic issues, but adjusted each component in pursuit of our goal to have a mathematically-informed civic dialogue. Three examples, with all materials, can be found here https://draft.mathindemocracy.education.uconn.edu/. The adaptations were effective in creating opportunities for participants to attend to the relevant quantitative aspects of civic issues, and to use that information and quantitative reasoning to support sense making and perspective taking. Specifically, our adjustments in Round 2 explicitly brought in the quantitative elements, and these were perhaps the most significant adjustment made.

The Mathematics and Important Mathematical Goals

Reflecting on our original purposes, our goals in developing this MinD model were to help reveal the central role of mathematics and quantitative reasoning in deliberating civic issues and to engage student participants in civic dialogue. We are mindful of our students as citizens and future voters who will have the opportunity to weigh in on a range of issues that can be informed by quantitative reasoning and mathematical analysis. In life, civic issues do not appear with mathematical tags or prior worked examples. Our commitments compelled us to frame the question in terms of the civic issues, create a context for inquiry by offering materials and sources, and then have the students author their approach to the problem. The math then was not prescribed but came from students deciding which mathematical tools would be useful in helping them gain new information and insight as they sought to develop their stance and make an informed decision. Although we did not "lead" with math content goals, if we are to consider using such lessons in schools to build capacity, we need to be able to articulate the mathematics learning that is intended or potentially occasioned by these lessons.

Mathematical practices

The mathematical work in the MinD lessons aligns with standards of mathematical practice (SMPs) from the US's Common Core State Standards (NGA, 2010), perhaps most specifically SMP1: Making sense of problems and persevering in solving them; and SMP5: Use appropriate tools strategically. These crucial practices—of making sense of a problem situation and deciding what mathematical tools are useful—are not typical of a school-math diet and are competencies students are unlikely to develop without deliberate attention and opportunity. Both of these practices require agency and independence on the part of the student. Stender and Kaiser (2015) note the importance of organizing education to develop this independence for students, "*so that they learn how realistic problems are really solved in the world around us, where nobody gives advice for the solution of real world examples nor simplifies the problems in such a way that they are solvable for the problem poser*" (p. 1255). For pedagogical purposes, as we developed this lesson model, we increased the structure and attention to bringing in mathematics explicitly, but we also aimed to maintain a high degree of agency and did not prescribe the mathematics.

Equally important, we find that two other standards of mathematical practice are consistently present when engaging the question *what should we do?* SMP3 is Construct viable arguments and critique the reasoning of others, and SMP 4 is Model with mathematics where mathematical modeling is "the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution" (Niss et al., 2007, p 12, as cited in Ostkirchen & Greefrath, 2022). These practices are required when examining and dialoguing about an issue from a mathematical lens where ultimately one must decide and act, ideally in just and fair ways.

As a consequence of these features, the MinD tasks are self-differentiating, and students can tackle the mathematical aspects of the modeling with different tools and different levels of sophistication. Indeed, Maaß (2005) observed that "the open formulation of modelling tasks and the necessity to simplify the complex reality allow the students to develop solutions depending on their capabilities" (p. 71). This consequence can be very beneficial to teachers and a school system as they strive to engage all their students in meaningful mathematical work.

It is important to point out a crucial feature of a MinD lesson, which contrasts with typical math lessons and math class. In the MinD lesson, students engage in constructing mathematical models and arguments *alongside their values and sense of what is right and fair*. These are intertwined as students determine their questions, contextualize the results of their mathematical work, and support their stances. The nature of this kind of reasoning—that doesn't narrow the possibilities for reasoning about a situation to primarily one disciplinary area—is not well understood, but it has been explored by Chorney and colleagues in mathematics (Chorney et al., 2024), and by other scholars in science (e.g., Sadler et al., 2007), and it is an important line of research to consider.

To summarize, the mathematical value of these lessons can be found in their alignment with established standards of mathematical practice; their pedagogical affordance in providing self-differentiating activities while maintaining engagement at a high level of cognitive demand; and their authentic nature and importance for future participation in society where students' values intertwine and shape their mathematical inquiry. Students found the work we did together to be very different from the work they did in their typical math classes and also different from the work they do in their social studies classes. Thus, the mathematical opportunities offered by these lessons are unique and powerful, as well as desperately needed for civic dialogue and the health of democratic societies.

Further Inquiry and Next Steps

In piloting these varied lessons, we became more aware of the unique nature of mathematical and civic reasoning that was brought to bear on these various issues. The MinD lesson model then offers another approach, complementing existing educational experiences for students and expanding possibilities for engaging civic issues within mathematics classrooms.

The design and implementation process we engaged to develop a MinD lesson model raised many questions. One compelling question for us is how, in our daily lives, do we engage quantitative information in reasoning about civic issues? We found we needed to create a math-focused activity to explore an issue (or part of the issue) in a mathematically-informed way. This requirement points to what seems to be a very

significant challenge for democratic dialogues in civic life. As people engage with topics that are math-infused or math-intensive in a democratic society, how do they identify and use (make sense of) the quantitative aspects of a topic? How could they learn, for example, what vaccine efficacy is?

Our original approach—which included readings and a protocol—was not sufficient for bringing out the quantitative information and using it in significant ways to make sense of an issue, gain new insights, or see new perspectives. Was that because people have little experience working in those ways? Was it because quantitative analyses are unlikely to happen through discussion and verbal processing, and rather require pen-to-paper, so to speak, or other technologies? Could we expect the general populous to do this kind of work in their lives as citizens? We would like to gain more insight into these questions.

We further wonder: Was the nature of activity we prompted in the MinD lessons similar to the practices of people who do use the quantitative information and analytic techniques for making sense of civic issues? Or rather, was the nature of the activity not very authentic, particularly as we got more "heavy handed" in designing the inquiry? Answering these questions, at least in part, would help us design better and more targeted educational experiences to ensure citizen students are prepared to engage in productive civic dialogue, informed from many perspectives.

We also reflect here on whether there are other sources and experiences that could support the development of these valuable capacities and practices. For example, during the pandemic, we noticed animations and graphics that sought to explain to citizens some of these key elements, perhaps most notably the idea of *flattening the curve* (Roberts, 2020; Stevens, 2020), when lockdowns were first instituted. Moving beyond our education system then, we wonder about an increased role for media, and perhaps our elected representatives, to be involved in identifying and helping citizens unpack critical quantitative aspects of civic issues. If quantitative aspects are common, how can citizens gain access to those ideas for use in deliberations? These questions are ones we would like to further explore, and we invite others to explore them as well.

Conclusion

In this paper, we have detailed the original Encounters protocol and its derivative, Collaborative Conversations, and how the MinD lesson model evolved as an adaptation of this protocol. Two core tenets drove the development of these lessons: (1) many societal issues require a mathematical lens to understand essential aspects of the issue, and (2) these quantitative elements should be part of the public discussion and deliberation about the issue. The MinD lesson model holds promise as an approach to support public discourse about civic issues and to support participants in getting better at engaging civic conversations.

As outlined in the opening of this paper, mathematics has much to contribute to democratic functioning and just decision making. As data generation and computing capacity continue to increase, and as societies' democratic norms are challenged, it seems increasingly important to build capacity among citizens to engage democratic dialogues to attend to and leverage quantitative information and reasoning. We hope there are opportunities in the mathematics education space and beyond to productively expand our collective capacity to advance us toward a more just society.

References

Ani, K. (2021). *Dear Citizen Math: How math class can inspire a more rational and respectful society*. Damascus Rodeo publishing.

Bartell, T. G. (2013). Learning to teach mathematics for social justice: Negotiating social justice and mathematical goals. *Journal for Research in Mathematics Education*, *44*(1), 129-163. https://doi.org/10.5951/jresematheduc.44.1.0129

Berry III, R. Q., Conway IV, B. M., Lawler, B. R., & Staley, J. W. (2020). *High school mathematics lessons to explore, understand, and respond to social injustice*. Corwin Press.

Brammer, L., Dumlao, R., Falk, A., Hollander, E., Knutson, E., Poehnert, J., Politano, A., & Werner, V. (2012). Core Competencies in Civic Engagement. Available at: https://scholarworks.merrimack.edu/ced_pubs/1

Cambridge Dictionary (n.d.). Justice. In *Dictionary.cambridge.org*. Retrieved January 2, 2023, from https://dictionary.cambridge.org/us/dictionary/english/justice

Chorney, S., Evans, K., & Staples, M. (2024). Conceptualizing reasoning practices in the context of sociomathematical issues. *Journal of Mathematical Behavior, 73*. Online and https://doi.org/10.1016/j.jmathb.2024.101124

Collins Dictionary (n.d.). Justice. In *Collinsdictionary.com*. Retrieved January 2, 2023, from https://www.collinsdictionary.com/us/dictionary/english/justice

Connected Mathematics Project (n.d.) Planning a lesson. Retrieved on January 3, 2022 from https://connectedmath.msu.edu/classroom/getting-organized/lesson.aspx.

Council of Europe (2018a). *Reference Framework for Competences in Democratic Culture. Context, Concepts and Models*. Volume 1. Council of Europe Publishing. Retrieved from https://rm.coe.int/prems-008318-gbr-2508-reference-framework-of-competences-vol-1-8573-co/16807bc66c

Council of Europe (2018b). *Reference Framework for Competences in Democratic Culture. Descriptors of competences for democratic culture*. Volume 2. Council of Europe Publishing. Retrieved from https://rm.coe.int/prems-008418-gbr-2508-reference-framework-of-competences-vol-2-8573-co/16807bc66d

Council of Europe (2018c). *Reference Framework for Competences in Democratic Culture. Guidance for implementation*. Volume 3. Council of Europe Publishing. Retrieved from https://rm.coe.int/prems-008518-gbr-2508-reference-framework-of-competences-vol-3-8575-co/16807bc66e

Dewey, J. (1923). *Democracy and education: An introduction to the philosophy of education*. Macmillan.

Evans, K., & Staples, M. (2021). Math and fairness: A key contribution of math to social justice. *Math Circle Network*. https://medium.com/math-circular/math-and-fairness-1c21bc45d5a1

Fischer, A., Duncombe, C., & Syverson, E. (2021). 50-State Comparison: K-12 and Special Education Funding. Education Commission of the States. https://www.ecs.org/50-state-comparison-k-12-and-special-education-funding/

Gibson, E., Johnston, M., Gooding, H., Dede, C., Phelps-Coco, A., Lin, K., Levinson, J.A., Austin, S.B. (2020). Delivering Case Method Teaching Through Online Platforms. *Educational Designer, 4(13).* Retrieved from: http://www.educationaldesigner.org/ed/volume4/issue13/article51/

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics education*, *34*, 37-73. https://doi.org/10.2307/30034699

Gutstein, E., & Peterson, B. (Eds.). (2005). *Rethinking mathematics: Teaching social justice by the numbers*. Rethinking Schools.

Hassan, T. (2015). *A classroom revolution: Reflections on Harkness learning and teaching*. Phillips Exeter Academy

Hess, D. (2008). Controversial issues and democratic discourse. In L. S. Levstik & C. A. Tyson (Eds.), *Handbook of research in social studies education*, (pp. 124-136).

Hess, D. E. (2002). Discussing controversial public issues in secondary social studies classrooms: Learning from skilled teachers. *Theory & Research in Social Education*, *30*(1), 10-41. https://doi.org/10.1080/00933104.2002.10473177

Hess, D., & Posselt, J. (2002). How High School Students Experience and Learn from the Discussion of Controversial Public Issues. *Journal of curriculum and supervision*, *17*, 283-314.

Humanities for All. (2022, September). The Encounters Series. https://humanitiesforall.org/projects/encounters

Kokka, K. (2019). Healing-informed social justice mathematics: Promoting students' sociopolitical consciousness and well-being in mathematics class. *Urban Education*, *54*, 1179-1209. https://doi.org/10.1177/0042085918806947

Krathwohl, D. (2002). A revision of Bloom's taxonomy: An overview. *Theory Into Practice*, *41*, 212–218. https://doi.org/10.1207/s15430421tip4104_2

Lamb, E. (2019, December 10). Connecticut Gov. Ned Lamont Proposes Truck Tolls. *Transport Topics*. https://www.ttnews.com/articles/connecticut-gov-ned-lamont-proposes-truck-tolls

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge university press.

Lee, C. D., White, G., & Dong, D. (2021). Educating for Civic Reasoning and Discourse. *National Academy of Education*.

Levine, P. (2016). The question each citizen must ask. *Educational Leadership, 73 (6)*, 31-34.

Maaß, K. (2005). Barriers and opportunities for the integration of modelling in mathematics classes: Results of an empirical study. *Teaching Mathematics and Its Applications, 24*(2-3), 61-74. https://doi.org/10.1093/teamat/hri019

Mallory, B. L., & Thomas, N. L. (2003). When the medium is the message: Promoting ethical action through democratic dialogue. *Journal of College and Character,* 4(9), 2 – 9. https://doi.org/10.2202/1940-1639.1366

National Governors Association. (2010). Common core state standards. *Washington, DC.* https://www.thecorestandards.org/wp-content/uploads/Math_Standards1.pdf

Nikpour, G. (2022, November 24). Iran's Football Team Has Already Won. Opinion piece, *The New York Times*. https://www.nytimes.com/2022/11/24/opinion/iran-protests-world-cup.html

Niss, M. (1994). Mathematics in society. In Biehler, R., Scholz, R. W., Strässer, R., & Winkelmann, B. (Eds.), *Didactics of mathematics as a scientific discipline*, *13*, 379-390. Springer Netherlands

O'Neil, C. (2016). *Weapons of math destruction: How big data increases inequality and threatens democracy*. Broadway books.

Ostkirchen, F., & Greefrath, G. (2022). Case study on students' mathematical modeling processes considering the achievement level. *Modelling in Science Education and Learning*, *15*(1), 137-150. https://doi.org/10.4995/msel.2022.16506

Roberts, S. (2020, March 27). Flattening the Coronavirus Curve: One chart explains why slowing the spread of the infection is nearly as important as stopping it. *New York Times*. https://www.nytimes.com/article/flatten-curve-coronavirus.html

Rogoff, B. (1997). Evaluating development in the process of participation: Theory, methods, and practice building on each other. In E. Ansel, K. A. Renninger, A. Renninger (Eds.), *Change and development: Issues of theory, method, and application*, 265-285.

Rosen, J. (2022, December 21). How Do You Protest in the Face of Censorship? An Empty Sign: In China, Russia and the United Kingdom, unmarked sheets of white paper have become a potent symbol of defiance. *New York Times*. https://www.nytimes.com/2022/12/21/magazine/white-paper-protests-censorship.html

Rudolph, J. L. (2014). Dewey's "science as method" a century later: Reviving science education for civic ends. *American Educational Research Journal*, *51*(6), 1056-1083. https://www.jstor.org/stable/24546711

Sadler, T. D., Barab, S. A., & Scott, B. (2007). What do students gain by engaging in socioscientific inquiry?. *Research in science education*, *37*(4), 371-391. https://doi.org/10.1007/s11165-006-9030-9

Seeto, K., Way, T., & Staples, M. (2022). CT's Educational Cost Sharing (ECS) Formula. In Evans & Staples (Eds.). *Connecting Mathematics & Social Justice: Lessons and Resources for Secondary Math Teachers*. https://sites.google.com/view/mtc4socialjustice/mtc4sj-book

Skovsmose, O. (2021). Mathematics and crises. *Educational Studies in Mathematics*, *108*(1), 369-383. https://doi.org/10.1007/s10649-021-10037-0

Smith, M. S, & Stein, M. K. (2011). *5 Practices for orchestrating productive mathematics discussion*. National Council of Teachers of Mathematics.

Stender, P., & Kaiser, G. (2015). Scaffolding in complex modelling situations. *ZDM, 47*, 1255-1267. https://doi.org/10.1007/s11858-015-0741-0

Stevens, H. (2020, March 14). Why outbreaks like coronavirus spread exponentially, and how to "flatten the curve." *Washington Post*. https://www.washingtonpost.com/graphics/2020/world/corona-simulator

Stitzlein, S. M. (2020). *Using Civic Participation and Civic Reasoning to Shape our Future and Education*. Paper commissioned for the UNESCO Futures of Education report.

Tate, W. F. (2013). Race, retrenchment, and the reform of school mathematics. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: Teaching social justice by the numbers, second edition* (pp. 42–51). Rethinking Schools.

Vigdor, N. (2019, March 8). Five reasons to support tolls and five reasons to oppose them in Connecticut. *Hartford Courant*. https://www.courant.com/2019/03/08/five-reasons-to-support-tolls-and-five-reasons-to-oppose-them-in-connecticut/

Wenger, E. (1999). *Communities of practice: Learning, meaning, and identity*. Cambridge university press.

Wolfram, C. (2020). *The math(s) fix: An education blueprint for the AI age*. Wolfram Media, Inc.

Xia, R. (2020, Aug 2). Manhattan Beach was once home to Black beachgoers, but the city ran them out. Now it faces a reckoning. *The Los Angeles Times*. https://www.latimes.com/california/story/2020-08-02/bruces-beach-manhattan-beach

Zhao, B. (2021). *Reforming Connecticut's Education Aid Formula to Achieve Equity and Adequacy across School Districts*. Federal Reserve Bank of Boston Research Data Report No. 21-1, Available at http://dx.doi.org/10.2139/ssrn.3832305

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Retrieved from: http://www.educationaldesigner.org/ed/volume5/issue18/article75/